December 13, 2007

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #3 2007-2008

Adviser: Score the contest using the answers or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. 2330 +340 3000 Note: 3 + 4 = 0 with one to carry. This occurs when the number base is 7. ANSWER: Base 7	1. Base 7
2. Let s = side of a square Each square has an area of 726 $cm^2 \div 6$ or 121 cm^2 $121 = s^2$ $\sqrt{121}$ = side of square = 11 Perimeter = 11 x 14 = 154 ANSWER: 154 cm	2. 154 cm
3. One third less than two = $2 - \frac{1}{3} = 1\frac{2}{3}$ One half less than three = $3 - \frac{1}{2} = 2\frac{1}{2}$ $2\frac{1}{2} - 1\frac{2}{3} = \frac{5}{6}$ ANSWER: $\frac{5}{6}$	3. $\frac{5}{6}$
4. Let x = people behind me Let x + 2 = people in front of me x + x + 2 + 1 = 3x (Don't forget "me") 2x + 3 = 3x 3 = x 5 = x + 2 ANSWER: 5 people ahead of me.	4. 5
5. Let p = original purchase price p - .25p = .75p .75p - .10(.75p) = .75p - .075p = .675p .675p - .03(.675p) = 209.52 .675p - .02025p = 209.52 .65475p = 209.52 p = $320 ANSWER: $320.00	5. $320.00
6. In S, let x = 1st integer x+x+1+x+2+x+3+x+4+x+5+x+6 = 7 7x + 21 = 7 7x = (-14) x = (-2) S {-2, -1, 0, 1, 2, 3, 4} In T, let y = 1st integer y+y+1+y+2+y+3+y+4+y+5+y+6+y+7 = (-12) 8y + 28 = (-12) y = (-5) T {-5, -4, -3, -2, -1 ,0, 1, 2} 5 integers (-2, -1, 0, 1, 2) are both in sets S and T ANSWER: 5	6. 5
7. A number is divisible by 6 if it is divisible by both 2 and 3. Out of the resulting outcomes, $\frac{2}{5}$ of the numbers will be divisible by 2. Out of the resulting outcomes, all the numbers are divisible by 3. (1 + 2 + 3 + 4 + 5 = 15) Therefore, $1(\frac{2}{5}) = \frac{2}{5}$ ANSWER: $\frac{2}{5}$	7. $\frac{2}{5}$

January 10, 2008

AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #4 2007-2008

Adviser: Score the contest using the answers or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. \$100 + 6% = \$106 (first year) \$106 + 100 = \$206 + 6% = \$218.36 (second year) \$218.36 + 100 = \$318.36 + 6% = \$337.46 (third year) \$337.46 + 100 = \$437.46 + 6% = \$463.71 (fourth year) \$463.71 + 100 = \$563.71 + 6% = \$597.53 (fifth year) ANSWER: \$597.53	1. \$597.53
2. Let x, x+1, x+2, x+3, x+4, x+5 = 6 consecutive numbers $x + x + 1 + x + 2 + x + 3 + x + 4 + x + 5 = 87$ $6x = 72$ $x = 12$ The numbers are 12, 13, 14, 15, 16, 17 ANSWER:12, 13, 14, 15, 16, 17	2. 12,13,14, 15,16,17
3. Let b = number of bananas eaten on day 1 $b + (b + 6) + (b + 12) + (b + 18) + (b + 24) = 100$ $5b = 40$ $b = 8$ The ape ate 8, 14, 20, 26 and 32 bananas on each of 5 days. ANSWER: 8, 14, 20, 26 and 32 in this order	3. 8, 14, 20, 26 and 32 in this order
4. Let A = amount of money Al had. Let B = amount of money Bill had $(A + 50) = 3(B - 50)$ $B + 30 = A - 30$ $B = A - 60$ (substituting) $A = 3(A - 60) - 200$ $190 = A$ ANSWER: 190	4. 190 or \$190
5. By adding, the Fibonacci sequence is: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 610 is the first term with 0 in the units' digit. ANSWER: 610	5. 610
6. Seek multiples of 31 in the 1800's $59 \cdot 31 = 1829$ but $1829 + 59 = 1888 < 1900$ (reject) $60 \cdot 31 = 1860$ and $1860 + 60 = 1920$. Edith was 40 years old in 1900 **OR** $61 \cdot 31 = 1891$ and $1891 + 61 = 1952$. Edith was 9 years old in 1900. (Reject: she is too young) ANSWER: 40 years old	6. 40 years old
7. $\text{Area}_{ABCD} = 6^2 = 36$ $\text{Area}_{XYA} = \frac{1}{2}(3 \cdot 4) = 6$ $\text{Area}_{BXC} = \frac{1}{2}(3 \cdot 6) = 9$ $\text{Area}_{CDY} = \frac{1}{2}(2 \cdot 6) = 6$ By exclusion, $\text{Area}_{XYC} = \text{Area}_{ABCD} - (\text{Area}_{XYA} + \text{Area}_{BXC} + \text{Area}_{CDY})$ $\text{Area}_{XYC} = 36 - (6 + 9 + 6)$ $\text{Area}_{XYC} = 36 - 21 = 15$ ANSWER: 15 square units or 15	7. 15 square units or 15